DYNAMICS OF A SHORT STEEL BAR WITH INHOMOGENEOUS PROPERTIES OF LAGGING CREEP ALONG ITS LENGTH

G. I. Popov

UDC 539.4.019

The equation of the mechanical state is taken in the stage of elastic deformation in accordance with Hooke's law, and the stage of elastoviscoplastic deformation, in differential form [1-3]:

$$\sigma = E\varepsilon \text{ with } t \leqslant t_{t.d}; \tag{1}$$

$$E\dot{\varepsilon} = \dot{\sigma} + \frac{\sigma_c}{\tau}\left[\exp\left(\frac{\sigma - f(\varepsilon)}{\sigma_c}\right) - 1\right] \text{with } t \geqslant t_{t.d}; \tag{2}$$

where $t_{t.d}$ is the time of the appearance of plastic deformation in the "weakest" cross section of the bar; σ_c, τ are constants; $f(\varepsilon)$ is the calculating static $\sigma - \varepsilon$ diagram:

$$f(\varepsilon) = E\varepsilon \text{ with } \varepsilon \leqslant \varepsilon_{s.t},$$
$$f(\varepsilon) = \sigma_t = E\varepsilon_{s.t} \text{ with } \varepsilon_{s.t} \leqslant \varepsilon \leqslant \varepsilon_{e.t},$$
$$f(\varepsilon) = \sigma_t + E_{hard}(\varepsilon - \varepsilon_{e.t}) \text{ with } \varepsilon_{e.t} \leqslant \varepsilon \leqslant \varepsilon_{unif};$$

E, E_{hard} are the moduli of elasticity and hardening; $\varepsilon_{s.t}$, $\varepsilon_{e.t}$, ε_{unif} are the deformations, corresponding, respectively, to the start of plastic deformations, the end of the area of creep, and the limiting uniform elongation.

With a relatively small range of deformation rates, for determination of the lag time of plastic deformation the Schmidt criterion can be used [4],

$$\int_0^{t_{lag}} \exp\left[\frac{\sigma(t_1) - \sigma_t}{\sigma_t m}\right] dt_1 = \tau_0, \tag{3}$$

where m, τ_0 are constants; t_1 is the time, calculated from the moment that the stresses attain the static yield point σ_t; t_{lag} is the lag time of plastic deformation, reckoned from the same moment.

Measurement of small plastic distributions of mild steel under dynamic loading has brought out extremely nonuniform distribution over the length of rods [5, 6]. In [7] it is noted that the recording of a falling (with respect to creep) "section of the diagram is actually impossible in the plastic region with existing methods of investigation, and the rate of deformation is found to be variable and to differ from the rate of deformation in the plastic region; therefore, the initial section of the curve of the plastic deformation cannot be regarded as reliable" and the "referral of the plastic resistance to the mean rate of deformation, on the chosen base of the measurement, can have only an arbitrary character."

The mechanism of the appearance of nonidentical plastic deformations can be interpreted using Eq. (2), if functional dependences instead of constant dependences are introduced into it. However, integration of this equation, for example, with variable (depending on the abscissa of the cross section and the time) values of the constants τ and σ_c is rather complicated and does not take in a transitional process, where part of the cross sections work elastically and the other part elastoplastically. An analysis of the development of test samples shows that the plasticity does not take in the whole volume of the sample immediately, but arises first in the "weakest" cross sections and is then propagated with a finite velocity to the "stronger" cross sections. The nonsimultaneous appearance of the plastic deformations brings about their nonuniform distribution over the length of the rod in the first stages of plastic deformation. The nonuniformity of the plastic deformations can, in a first approximation, be due to the nonsimultaneous appearance of the plastic deformations, i.e., the constants m and τ_0 in the universal criterion of the creep (3) can be taken as functions of the abscissa of the cross section x.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 149-154, November-December, 1977. Original article submitted November 9, 1976.

 0021-8944/77/1806-0852$07.50

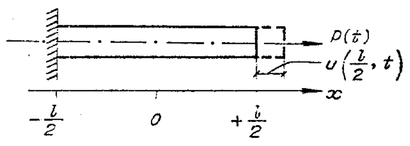

Fig. 1

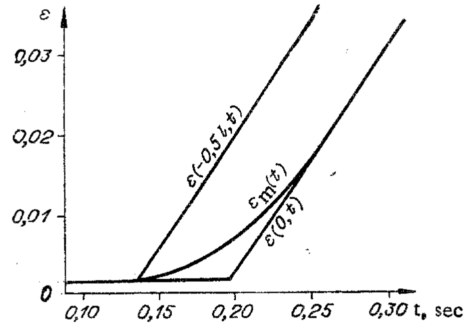

Fig. 2

Analysis of the experimental data showed that we can limit ourselves to the change of only one coefficient m. The universal criterion of the creep can now be written in the following manner:

$$\frac{1}{\tau_0}\int\limits_0^{t_{lag}(x)} \exp\left[\frac{\sigma(t_1)-\sigma_t}{m(\xi)\,\sigma_t}\right]dt_1 = 1, \tag{4}$$

where $m(\xi)$ is a function of the abscissa of the cross section.

Using the criterion (4), Eq. (2) must be integrated for each cross section under its initial conditions.

The dependence $m(\xi)$ is chosen on the basis of the experimental data, taking account of different loading conditions, for the "weakest" (m_{min}) and "strongest" (m_{max}) cross sections. The value of the coefficient m was interpreted as a random quantity, obeying a Gaussian distribution; the area of the asymptotic parts of the curve of the normal distribution beyond the limits of the segment (m_{min}, m_{max}) was a determined part of the area of the whole curve. The "weakest" cross section (m_{min}) was arbitrarily referred to the left-hand end of the rod with the abscissa $(-l/2)$ and the "strongest" (m_{max}) to the right-hand end with the abscissa $(+l/2)$.

The expression for $m(\xi)$, obtained after integration of the curve of the Gaussian distribution, is cumbersome and inconvenient for further calculations. The following approximation is sufficiently exact and simple:

$$m(\xi) = m(0)\,[1 + k_1\xi - k_3\xi^3], \tag{5}$$

where $m(0) = m_m = (m_{min} + m_{max})/2$ is the value of m for the central cross section of the bar; $\xi = x/l$; k_1 and k_3 are coefficients, determined from the boundary conditions

$$m(-0.5) = m_{min},\quad m(+0.5) = m_{max}.$$

Using (1), (2), (4), and (5) it is possible to solve various problems (in a quasistatic statement) of the elongation of a steel bar, whose properties of lagging creep are not constant along the length. In view of the small length of the bar l and the relatively high rate of change in the stresses, we neglect wave processes in the bar. We assume the left-hand side of the bar to be fixed and the right-hand side to be free (Fig. 1). From what has been said it follows that the stresses along the length of the bar are constant,

$$\sigma(x, t) = \sigma(t), \tag{6}$$

and the expression for the displacements has the form

$$u(x, t) = \int\limits_{-l/2}^{x} \varepsilon(x, t)\,dx.$$

The time of the appearance of creep in the cross section x, reckoned from the start of the loading, is equal to

$$t_{t.d}(x) = t_t + t_{lag}(x).$$

Equation (2), taking account of (6), is written in the form

$$E\,\frac{\partial\varepsilon(x, t)}{\partial t} = \frac{d\sigma}{dt} + \frac{\sigma_c}{\tau}\left[\exp\left(\frac{\sigma - f(\varepsilon)}{\sigma_c}\right) - 1\right].$$

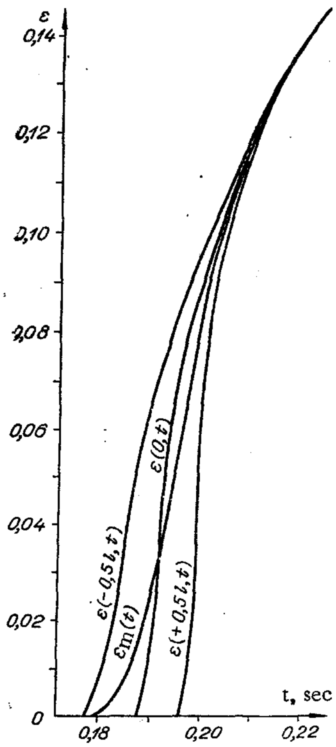

Fig. 3

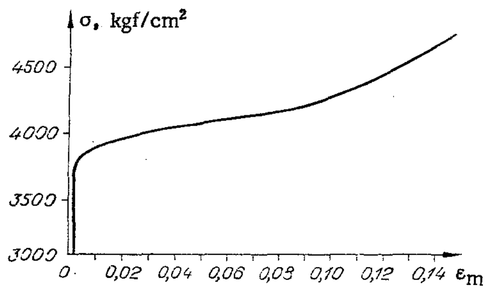

Fig. 4

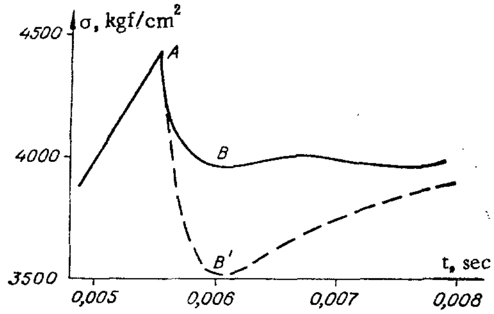

Fig. 5

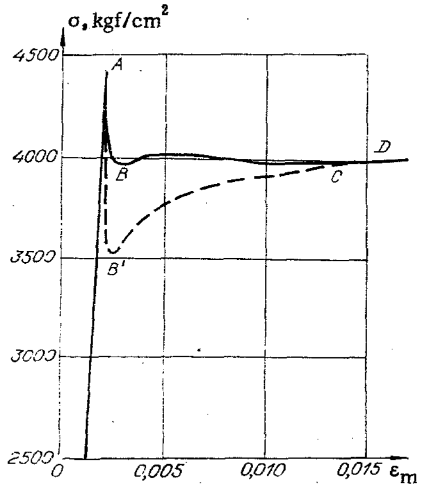

Fig. 6

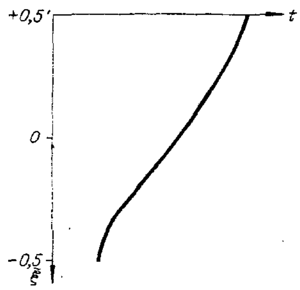

Fig. 7

Let us consider the four following problems:

1) Determine the deformations $\varepsilon(x, t)$ and displacements $u(x, t)$ of the cross sections of a bar with the sudden imposition of the force $P(t) = F\sigma(t)$, invariable with time, where F is the area of the transverse cross section of the bar. and the stress $\sigma(t) = \sigma = \text{const}$ exceeds the static yield point σ_t;

2) determine $\varepsilon(x, t)$ and $u(x, t)$ with the action of a linear force $P(t)$, rising according to a linear law

$$P(t) = E\alpha Ft, \text{where } \alpha = \text{const};$$

3) determine $\sigma(t)$, $\varepsilon(x, t)$, $u(x, t)$ with a displacement of the free end of the bar with a constant velocity $u(l/2, t) = \alpha l t$;

4) determine $\sigma(t)$, $\varepsilon(x, t)$, $u(x, t)$ with a displacement of the free end of the bar: in the elastic stage with a constant velocity and, in the elastoplastic stage, with the acceleration:

$$u(l/2, t) = \alpha l t \text{ with } t \leqslant t_{t.d}(-l/2),$$
$$u(l/2, t) = \alpha l t + \beta l[t - t_{t.d}(-l/2)]^2 \text{ with } t \geqslant t_{t.d}(-l/2), \text{ where } \beta = \text{const}.$$

With solution of problems 1 and 2, knowing the law of change of the stresses with time $\sigma(t)$, from (4), (5) we can obtain an equation for determining the abscissa $\tilde{x}$ of the boundary of the zones of elastic and elastoviscoplastic deformation of the material. In problems 3 and 4, this law is unknown; therefore, the motion of the boundary of the zones $\tilde{x}$ can be found only by solving a system of equations.

The solution of problems in an M-220 digital computer (the ALGOL program was constructed by V. V. Samarin) was carried out with the following values of the constants:

$$\sigma_t = 2607 \text{ kgf/cm}^2,\ E = 2.1 \cdot 10^6 \text{ kgf/cm}^2,\ \tau_0 = 1 \text{ sec},\ m(0) = 0.092,\ k_1 = 0.160,\ k_3 = 0.106,$$
$$\sigma_c = 275 \text{ kgf/cm}^2,\ \tau = 0.00262 \text{ sec},\ \varepsilon_{e.t} = 0.04,\ \varepsilon_{unif} = 0.15,\ E_{hard} = 16{,}600 \text{ kgf/cm}^2.$$

Figure 2 (problem 1, $\sigma = 1.2\sigma_t = \text{const}$) shows that taking account of the properties of the "weak" cross section [line $\varepsilon(-l/2, t)$], determining the start of creep, or the properties only of the "strong" cross section [line $\varepsilon(+l/2, t)$], determining the moment when the creep takes in the whole bar, or only of the central cross section [line $\varepsilon(0, t)$], does not give a correct representation of the mean deformation $\varepsilon_m(t)$; it can be seen that the lines $\varepsilon_m(t)$ and $\varepsilon(0, t)$ practically coincide only with $\varepsilon \geq 0.016$.

For the conditions $\dot{\sigma} = \text{const}$ (problem 2, $\alpha = 0.01$), the lines $\varepsilon_m(t)$ and $\varepsilon(0, t)$ intersect only with $\varepsilon \cong 0.03$ (Fig. 3). Figure 4 shows the initial part of the theoretical $\sigma - \varepsilon$ diagram for steel St.3; the form of the curve is in good agreement with the experimental data of [8].

The curves of $\sigma(t)$ and $\sigma(\varepsilon)$ (Figs. 5, 6) were plotted for the conditions $\dot{\varepsilon} = \text{const}$ within the limits of the elasticity and the accelerated deformation in the elastoplastic region (problem 4). The dashed line shows the curves of $\sigma(t)$ and $\sigma(\varepsilon)$ plotted under the assumption that the creep takes in the whole sample immediately (in accordance with Kelly) and that there is a drop in the stresses from the upper yield point to the lower in accordance with a functional dependence (line AB') having a "memory." These curves differ sharply from the experimental. A shortcoming of the theoretical solution proposed in [3], leading to an enormous difference between the upper and lower yield points (Fig. 6), can be eliminated by taking account of the nonsimultaneous creep of the material along the length of a sample. An analysis of the example given shows that, in approximate calculations, the mean value of the "modulus of the relaxation" for mild steel (brand St.3) can be taken equal to $E_{rel} = -10^6$ kgf/cm^2.

In Fig. 7 it is possible to follow the motion with time of the abscissa $\tilde{\xi} = \tilde{x}/l$ of the boundary between the zones of elastic and elastoviscoplastic deformation of the material. The creep takes in the whole sample only ≈ 0.008 sec after the start of loading.

The use of the equation of the mechanical state in differential form, at the same time taking account of the inhomogeneity of the lagging creep, gives significant refinements of the calculation with deformations of the material not exceeding $\varepsilon = 0.003$-0.005.

LITERATURE CITED

1. V. V. Sokolovskii, "The propagation of elastoviscoplastic waves in bars," Prikl. Mat. Mekh., 12, No. 3 (1948).
2. L. E. Malvern, "The propagation of longitudinal waves of plastic deformation in a bar of material exhibiting a strain-rate effect," J. Appl. Mech., No. 18 (1951).

3. J. M. Kelly, "Strain-rate sensitivity and yield point behavior in mild steel," Int. J. Solids Struct., No. 3 (1967).
4. R. C. Smith, T. E. Pardue, and J. Vigness, "The mechanical properties of certain steels as indicated by axial dynamic-load tests," Proc. SESA, No. 23 (1956).
5. A. Nadai, Plasticity and the Fracture of Solids [Russian translation], IL, Moscow–Leningrad (1954).
6. E. V. Lomakin and A. F. Mel'shanov, "The behavior of low-carbon steels with elongation," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 4 (1971).
7. Yu. N. Rabotnov, "Model of an elastoplastic medium with a lag of the creep," Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1968).
8. E. A. Davis, "Influence of rate of strain and temperature on yield stress of mild steel," J. Appl. Mech., Dec. (1944).

TIME CRITERIA OF EXPLOSIVE FRACTURE

Yu. I. Fadeenko

UDC 539.375

The total fracture of a solid in a given section presupposes the satisfaction of the following time criteria: 1) the fracture preparation criterion (damage accumulation, formation of embryonic cracks); 2) the integral crack coalescence criterion, based on the nonstationary crack growth equation.

In solving specific problems it may prove convenient to consider separately the initial (essentially nonstationary) phase of acceleration of cracks initially at rest and the subsequent phase of quasistationary growth; in this case the second of the above-mentioned criteria breaks down into two separate time conditions. The starting relations may also include Griffith's criterion, i.e., a differential crack growth condition requiring that the energy-release rate be not less than the work-absorption rate. Generally speaking, Griffith's criterion should be obtained from the crack growth equation by equating the growth rate to zero.

Thus, the total fracture time τ can be represented as the sum of the fracture preparation time τ_1, the duration of the transient process τ_2, and the period of quasistationary growth leading to total coalescence of the cracks τ_3:

$$\tau = \tau_1 + \tau_2 + \tau_3. \tag{1}$$

In recent years the kinetic theory of fracture has gained wide acceptance. The fundamental principles of the kinetic theory have received extensive experimental confirmation; for alloys and polymers they have proved to be so general that deviations from them have been the subject of special investigation. However, the experiments on which the theory is based relate to the region of large rupture lives (10^{-3} sec and more). Until recently it was uncertain whether the kinetic theory could be applied on the interval of short rupture lives (10^{-6} sec or less) typical of explosive fracture. Here it is shown that the region of applicability of the kinetic theory, as usually formulated, is limited and that on the interval of short rupture lives it should be substantially modified.

The basic relation of the kinetic theory – the time fracture criterion determining the rupture life τ [see (1)] of a solid subjected to the action of a constant tensile stress σ – is usually written in the following form:

$$\tau = \tau_0 \exp \frac{u - \gamma\sigma}{kT}, \tag{2}$$

where k is Boltzmann's constant; T is temperature; τ_0 is the preexponential coefficient, which coincides in order of magnitude with the period of the thermal vibrations of the atoms (10^{-13}–10^{-12} sec); u is the activation energy (of the order of the atomic bond energy in the solid).

The factor γ is a characteristic of the actual processes preparatory to fracture that take place at the atomic level. It is usually assumed that γ characterizes the most dangerous of the structural defects – the microstress raisers; the quantity γ, which has the dimension of volume, can be interpreted as the product of the volume of the defect and the stress-concentration factor.

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 154-159, November-December, 1977. Original article submitted March 17, 1977.

 0021-8944/77/1806-0856$07.50